

Gradient-Free Aeroacoustic Shape Optimization Using Large Eddy Simulation

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We present an aeroacoustic shape optimization framework that relies on high-order flux reconstruction, the gradient-free Mesh Adaptive Direct Search optimization algorithm, and large eddy simulation. Our parallel implementation ensures consistent runtime for each optimization iteration, regardless of the number of design parameters, provided that sufficient resources are available. The objective is to minimize the overall sound pressure level (OASPL) at a near-field observer by computing it directly from the flowfield. We evaluate this framework across three problems. First, an open deep cavity is considered at a freestream Mach number of $M_\infty = 0.15$ and Reynolds number of $Re = 1500$, reducing the OASPL by 12.9 dB. Next, we considered tandem cylinders at $Re = 1000$ and $M_\infty = 0.2$, achieving over 11 dB of noise reduction by optimizing cylinder spacing and diameter ratio. Lastly, a baseline NACA0012 airfoil at $Re = 23,000$ and $M_\infty = 0.2$ is optimized to generate a new four-digit NACA airfoil at an appropriate angle of attack to minimize the OASPL while ensuring the baseline time-averaged lift coefficient is maintained and prevents any increase in the baseline time-averaged drag coefficient. The OASPL and mean drag coefficient are reduced by 5.7 dB and more than 7%, respectively. These results highlight the feasibility and effectiveness of our aeroacoustic shape optimization framework.

particularly in transient or highly dynamic environments. Hence, MADS emerges as a compelling choice for optimizing unsteady problems, offering a versatile and robust approach capable of navigating the complexities of time-dependent simulations. The suitability of MADS, coupled with HORUS, has been demonstrated in the works of Karbasian and Vermeire [33] and Aubry et al. [34] for aerodynamic shape optimization, and by Hamedi and Vermeire [35] for laminar aeroacoustic shape optimization.

In this study, we present an aeroacoustic shape optimization framework based on the FR approach and the gradient-free MADS optimization algorithm for LES. Building upon our prior work [35], which assessed this framework for two-dimensional problems at low Reynolds numbers, we extend its application to three dimensions. To the best of our knowledge, no other studies have integrated the gradient-free MADS optimization with a high-order LES solver. One significant limitation of this framework is its runtime because a high-order LES is performed for each objective function evaluation. The runtime of the optimization problem scales with the number of design parameters, requiring a corresponding number of CFD simulations in each optimization iteration. However, we addressed this challenge by implementing the optimization framework in parallel. This enables concurrent evaluation of candidate designs within each optimization iteration, effectively reducing the runtime of each iteration to that of a single CFD simulation and independent of the number of design parameters, provided that sufficient computing resources are available. Additionally, each CFD simulation is performed in parallel on state-of-the-art clusters using Graphical Processing Units (GPUs), highlighting the two-layer parallelism of the proposed optimization algorithm.

This paper is outlined as follows. The methodology is given in Sec. II. Then, the shape of a three-dimensional open cavity is optimized to reduce noise in Sec. III, followed by three-dimensional tandem cylinders in Sec. IV, and, airfoil shape optimization for noise reduction is performed in Sec. V. Finally, the conclusions and recommendations for future work are given in Sec. VI.

II. Methodology

This section presents an overview of the methodology employed to solve the unsteady Navier–Stokes equations, along with the aeroacoustic shape optimization framework.

A. Governing Equations

The compressible unsteady Navier–Stokes equations can be cast in the following general form:

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot \mathbf{F} = 0 \quad (1)$$

where t is time and $\mathbf{u}$ is a vector of conserved variables:

$$\mathbf{u} = \begin{bmatrix} \rho \\ \rho u_i \\ \rho E \end{bmatrix} \quad (2)$$

where ρ is density, ρu_i is a component of the momentum, u_i are velocity components, and ρE is the total energy. The inviscid and viscous Navier–Stokes fluxes are

$$\mathbf{F}_{\text{inv},j}(\mathbf{u}) = \begin{bmatrix} \rho u_j \\ \rho u_i u_j + \delta_{ij} p \\ u_j (\rho E + p) \end{bmatrix} \quad (3)$$

and

$$\mathbf{F}_{\text{vis},j}(\mathbf{u}, \nabla \mathbf{u}) = \begin{bmatrix} 0 \\ \tau_{ij} \\ -q_j - u_i \tau_{ij} \end{bmatrix} \quad (4)$$

respectively, where δ_{ij} is the Kronecker delta. The pressure is determined via the ideal gas law as

$$p = (\gamma - 1)\rho \left(E - \frac{1}{2} u_k u_k \right) \quad (5)$$

where $\gamma = 1.4$ is the ratio of the specific heat at constant pressure, c_p , to the specific heat at constant volume, c_v . The viscous stress tensor is

$$\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right) \quad (6)$$

and, the heat flux is

$$q_j = -\frac{\mu}{Pr} \frac{\partial}{\partial x_j} \left(E + \frac{p}{\rho} - \frac{1}{2} u_k u_k \right) \quad (7)$$

where μ is the dynamic viscosity and $Pr = 0.71$ is the Prandtl number.

B. Optimization Framework

In this study, we employed the minimal bases construction of the MADS optimization technique, similar to our previous work [35], for the open deep cavity and tandem cylinders. However, for the NACA0012 problem, we developed a parallel optimization framework, employing the maximal bases construction of the OrthoMADS algorithm [32], to address the runtime challenges inherent in serial implementation.

The flowchart of the proposed aeroacoustic shape optimization is illustrated in Fig. 1. The process begins by evaluating the baseline objective function, $\mathcal{F}_0$, with the incumbent design set equal to the baseline design, $\mathcal{I}_0 = \mathcal{F}_0$. The optimization algorithm then takes as inputs the problem's constraints, baseline design parameters $\mathcal{X}_0$, initial mesh size parameter Δ_0^m , and the baseline objective function $\mathcal{F}_0$. The mesh size parameter $\Delta^m \in \mathbb{R}_+$ defines the resolution of the design space $\mathcal{D}$, and it guides the selection of design candidates within each optimization iteration along with the poll size parameter Δ^p . The OrthoMADS algorithm employs polling directions orthogonal to each other, generating minimal convex cones of unexplored directions at each iteration, thus enhancing the efficiency of design space exploration [32]. Along with Fig. 1, Algorithm 1 delineates the parallel implementation of the OrthoMADS algorithm.

For the k th optimization iteration, candidate designs are identified, and an automated script generates the mesh for each geometry, with wall surfaces controlled by design parameters that directly influence the geometry. The HORUS is then called to compute the flowfield using high-order LES. The objective functions of the candidate designs, $\mathcal{F}_k^i$, are evaluated as a postprocessing step and compared to the incumbent design I_k . By comparing the objective function of these designs with the incumbent design, both the mesh size parameter and the incumbent design are updated, initiating the next optimization iteration. The optimization process stops when the mesh size parameter falls below 10^{-6} and the changes in design parameter values between two consecutive iterations are less than 1%. These criteria indicate the algorithm has successfully converged to an optimal design.

Notably, the for loop in lines 18–21 of Algorithm 1, which corresponds to the highlighted parts of Fig. 1, is the most computationally intensive part of the algorithm where a total of $2n$ CFD simulations are conducted, where n is the number of design parameters. Typically, each CFD simulation runs in parallel, and candidate designs are executed sequentially. However, in the proposed parallel implementation of the algorithm, all candidate designs run concurrently, reducing the runtime of $2n$ CFD simulations to that of a single CFD simulation, provided that adequate computer resources are available.

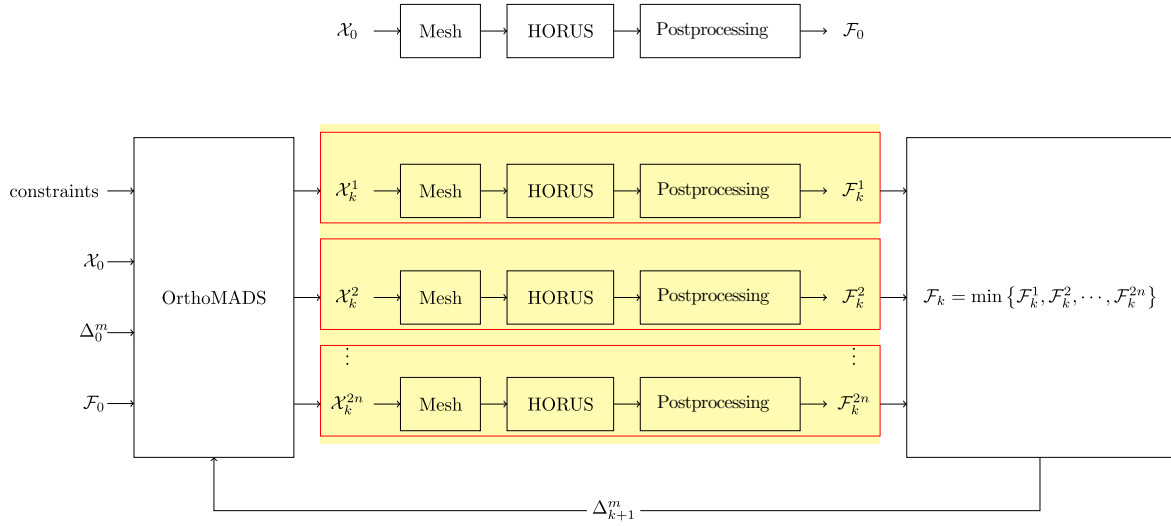


Fig. 1 Flowchart of the proposed aeroacoustic shape optimization framework.

Algorithm 1 The aeroacoustic shape optimization framework

```

1   $k = 0$ ;
2  MADS Iteration, iter = 0;
3  Run Baseline Design;
4  Evaluate  $\mathcal{F}_0$ ;
5  Define Incumbent  $\mathcal{I}_0 = \mathcal{F}_0$ ;
6  Define  $\Delta_0^m$ ;
7  while True do
8    if  $\Delta_k^m > \Delta_0^m$  then
9       $\Delta_k^m = \Delta_0^m$ ;
10   end
11   if minimal positive basis construction then
12      $\Delta_k^p = n \sqrt{\Delta_k^m}$ ;
13   end
14   if maximal positive basis construction then
15      $\Delta_k^p = \sqrt{\Delta_k^m}$ ;
16   end
17   Identify Candidate Designs,  $p_k^1, \dots, p_k^n$ ;
18   for  $i = 1, \dots, n$  do
19     Run HORUS for  $p_k^i$ ;
20     Evaluate  $\mathcal{F}_k^i$ ;
21   end
22   if  $\min\{\mathcal{F}_k^1, \dots, \mathcal{F}_k^n\} < \mathcal{I}_{\text{iter}}$  then
23      $\Delta_{k+1}^m = 4\Delta_k^m$ ;
24     iter += 1;
25      $\mathcal{I}_{\text{iter}} = \min\{\mathcal{F}_k^1, \dots, \mathcal{F}_k^n\}$ ;
26   else
27      $\Delta_{k+1}^m = (1/4)\Delta_k^m$ ;
28   end
29    $k += 1$ ;
30   if  $\Delta_k^m < 10^{-6}$  and  $|\mathcal{X}_k - \mathcal{X}_{k-1}/\mathcal{X}_{k-1}| < 0.01$  then
31     break;
32   end
33 end

```

C. Flow Solver

The in-house solver, HORUS, is utilized for solving the Navier–Stokes equations, employing the FR approach for spatial discretization. This approach is used to discretize the divergence operator for general advection–diffusion equations of the form shown in Eq. (1). It is a high-order accurate numerical method first introduced by Huynh [20] in 2007, and extended to multidimensions for mixed element types by Wang and Gao [36]. FR is appealing due to its accuracy, generality, robustness, and suitability for modern hardware architectures [18]. Compared to commonly used

low-order numerical methods, FR provides more accurate solutions using fewer degrees of freedom and at reduced computational cost [37]. We explained the FR approach in more detail in our previous work [35]. In this study, the second-order accurate Nasab–Pereira–Vermeire scheme [38], which incorporates an adaptive time-stepping method [39], is used to advance the solution in time.

III. Deep Cavity

Flow over an open deep cavity is a classical problem in fluid mechanics and aeroacoustics, and has been the subject of extensive research due to its relevance for a range of engineering applications. Such flows represent simplified versions of the complex dynamics over panel gaps, like those between windows/doors and the fuselage or between control surfaces and wings. These gaps profoundly influence aerodynamics, structural integrity, and noise levels of aircraft. The flow over a cavity is characterized by a complex interplay between the boundary layer, the recirculation zone inside the cavity, and the external flow. The occurrence of self-sustained oscillations of velocity and pressure can induce acoustic noise or strong vibrations. The presence of the cavity can lead to a variety of aerodynamic and aeroacoustic phenomena, such as flow separation, unsteady vortex shedding, and acoustic resonance. Understanding the aerodynamic and aeroacoustic characteristics of flow over a cavity is crucial for optimizing the design and performance of many engineering systems.

Extensive research has been conducted on two-dimensional cavity flows, leading to favorable agreement between experimental data and numerical two-dimensional simulations. Although three-dimensionality is observed in cavity flow experiments, it underscores the significance of conducting three-dimensional cavity flow simulations [40,41]. Lawson [42] reviewed the experimental and numerical studies of open cavities. Furthermore, the radiated noise from a cavity is studied via LES by several researchers [43–46]. The geometry of a three-dimensional cavity is usually given in terms of length-to-depth, L/D , and width-to-depth, W/D , ratios, as depicted in Fig. 2. In this section, flow over an open cavity is validated and then the noise at a near-field observer is minimized via the proposed gradient-free shape optimization framework.

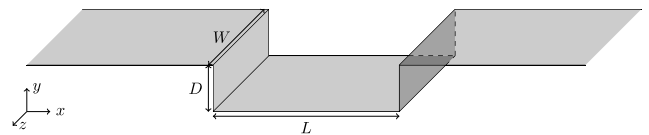


Fig. 2 The geometry of the three-dimensional open deep cavity.

A. Validation

In this section, we extend our previous work [35] by extruding it in the z direction. The grid convergence study is performed using the time-averaged drag coefficient, and overall sound pressure level (OASPL) measured at an observer located $7.16D$ above the cavity's center.

1. Computational Details

To be consistent with [35], the open cavity with a length-to-depth ratio of $L/D = 4$ is extruded in the z direction with a width-to-depth ratio of $W/D = 3$. The Reynolds number, based on the depth of the cavity, is $Re_D = 1500$, and the Mach number is 0.15. To ensure wake mode oscillations, the inlet boundary is placed $5D$ upstream of the cavity inlet, resulting in a boundary-layer thickness of $\delta/D \approx 0.2$ at the entrance of the cavity. The outflow boundary is placed $60D$ downstream of the cavity's trailing-edge wall, with the last $50D$ acting as a buffer region to eliminate acoustic wave reflections. The computational domain extends to $15D$ in the y direction with the last $5D$ as a buffer region. The grid stretching ratio is 1.05 and 1.075 for the resolved and buffer regions, respectively, with a minimum element size of $0.2D$ inside the cavity. A total of 14,652 hexahedral elements are used. The geometry and mesh of the three-dimensional cavity are shown in Figs. 2 and 3, respectively. The periodic boundary condition is used in the spanwise direction, no-slip boundary conditions are applied at the walls, and Riemann invariant boundary conditions are applied at the inlet and outlet of the computational domain. The simulation is run for $100t_c$, where $t_c = D/U_\infty$, to allow initial transients to disappear and then run for another $400t_c$ to average the statistical quantities. To ensure uncorrelated turbulent fluctuations at a separation of half the domain size, the correlation coefficient of the x component of the velocity perturbation along with that of the pressure perturbation are computed along the spanwise direction and depicted in Fig. 4. The results of the grid independence study are given in the next section.

2. Results and Discussion

The grid independence study is performed by increasing the solution polynomial degree, which increases the resolution of the simulation. The time-averaged drag coefficient and the OASPL at an observer located $7.16D$ above the center of the cavity are computed using solution polynomial degrees of $\mathcal{P}2$, $\mathcal{P}3$, and $\mathcal{P}4$ to show the grid independency.

The drag coefficient is defined as

$$C_D = \frac{F_x}{(1/2)\rho_\infty U_\infty^2 DW} \quad (8)$$

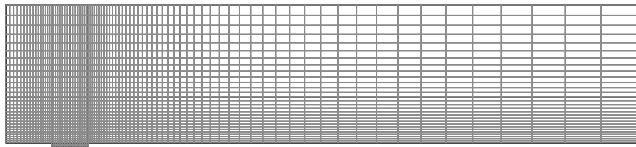


Fig. 3 The mesh of the three-dimensional open deep cavity.

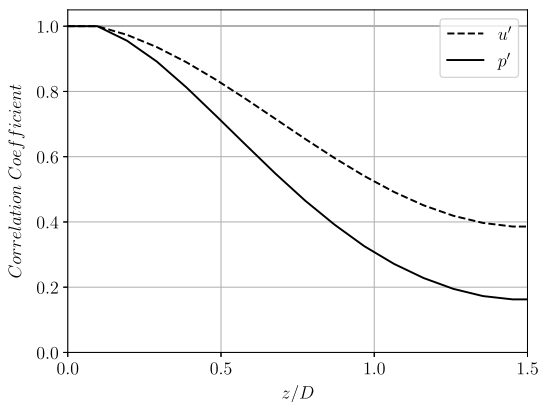


Fig. 4 The correlation coefficient in the spanwise direction for the three-dimensional open deep cavity.

where F_x is the force in the x direction computed on the three cavity walls, ρ_∞ is the freestream density, and U_∞ is the freestream velocity. Furthermore, the OASPL is computed using the following equation:

$$\text{OASPL} = 20 \log \left(\frac{p_{\text{RMS}}}{p_{\text{ref}}} \right) \quad (9)$$

where p_{RMS} represents the root-mean-square of the pressure perturbations, defined as

$$p_{\text{RMS}} = \sqrt{\frac{\sum_{i=1}^n (p'_i)^2}{n}} \quad (10)$$

where n is the total number of time samples, and the pressure perturbation p' is given by

$$p' = p - \bar{p} \quad (11)$$

with $\bar{p}$ being a vector of cumulative time-averaged pressure signals, each element of which is defined as

$$\bar{p}_k = \frac{\sum_{i=1}^k p_i}{k}, \quad k = 1, 2, \dots, n \quad (12)$$

Note that in this formulation, $\bar{p}$, p' , and p_{RMS} are vectors, where each element k corresponds to the calculation using k time samples. In Eq. (9), p_{RMS} refers to the last element of the p_{RMS} vector, that is, $p_{\text{RMS}} = p_{\text{RMS},n}$.

The time-averaged drag coefficient along with the OASPL at the observer, for different simulations, are given in Table 1. Thirty observer points along the span of the cavity are used. The time-averaged pressure and root-mean-square of the pressure perturbation are computed for each observer point and then spatially averaged to find the OASPL at the observer location. These results show that the $\mathcal{P}3$ simulation provides sufficient resolution for this study.

B. Optimization

In this section, the noise at the observer point located at $x_{\text{obs}}/D = [2, 7.16]$ is minimized by changing the height of the cavity trailing-edge wall, h_{TE} , depicted in Fig. 5. There are other possible shape parameters to minimize the cavity's noise, such as the length-to-depth ratio. The choice of the design parameter for shape optimization depends on the noise generation mechanism of interest. For instance,

Table 1 Summary of grid independence study of the open deep cavity

Simulation	$\bar{C}_D$	OASPL, dB
$\mathcal{P}2$	0.1314	112.1
$\mathcal{P}3$	0.1098	113.1
$\mathcal{P}4$	0.1115	113.3



Fig. 5 The design variable, h_{TE} , for the open deep cavity.

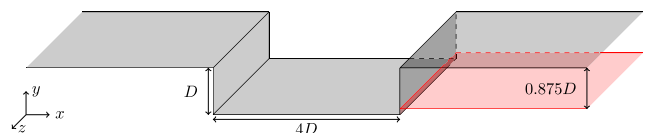


Fig. 6 The baseline, in black, and optimum, in red, designs of the open cavity.

if the focus is on exploring the shear layer extending over the cavity without vortex roll-up as the primary mechanism for sound generation, the length-to-depth ratio would be a more suitable design parameter. However, in this study, the focus is on determining whether the vortices become trapped within the cavity or rest on the downstream wall, thereby forming a backward-facing step. Thus, $\mathcal{X} = h_{TE}$ is the design variable and $\mathcal{X}_0 = 0$, while the objective function is $\mathcal{F} = p'_{rms}$. Upper and lower bounds of -1 and 4 , respectively, are chosen for the design variable, h_{TE} .

1. Results and Discussion

The optimization procedure converged after 19 MADS iterations with a total of 36 objective function evaluations. The optimal design parameter is identified as $h_{TE} = -0.875$, resulting in an OASPL of 100.3 dB, signifying a 13.0 dB reduction in noise. The baseline and optimum designs, depicted in Fig. 6, illustrate a notable reduction in emitted noise by lowering the trailing-edge wall of the cavity. However, such a modification may pose feasibility challenges in engineering applications. Moreover, Fig. 7 illustrates the explored

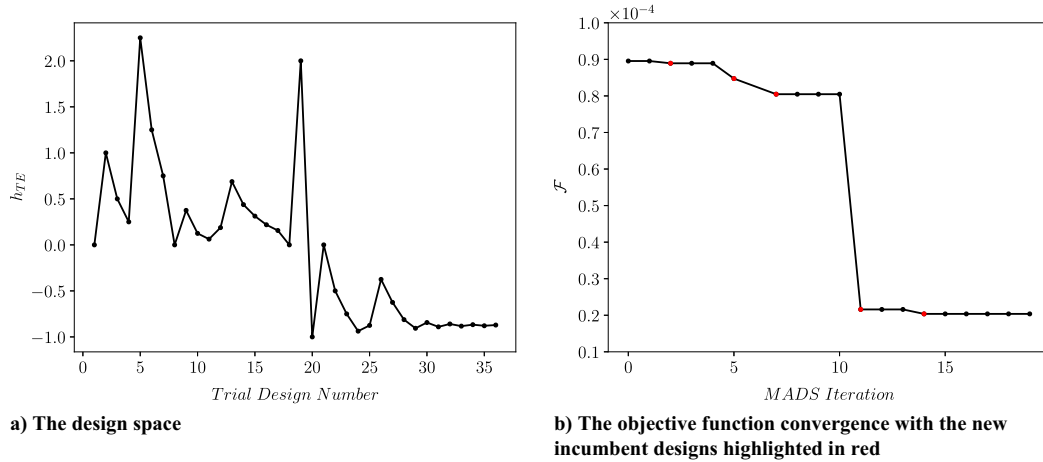


Fig. 7 The design space and objective function convergence for the three-dimensional open deep cavity.

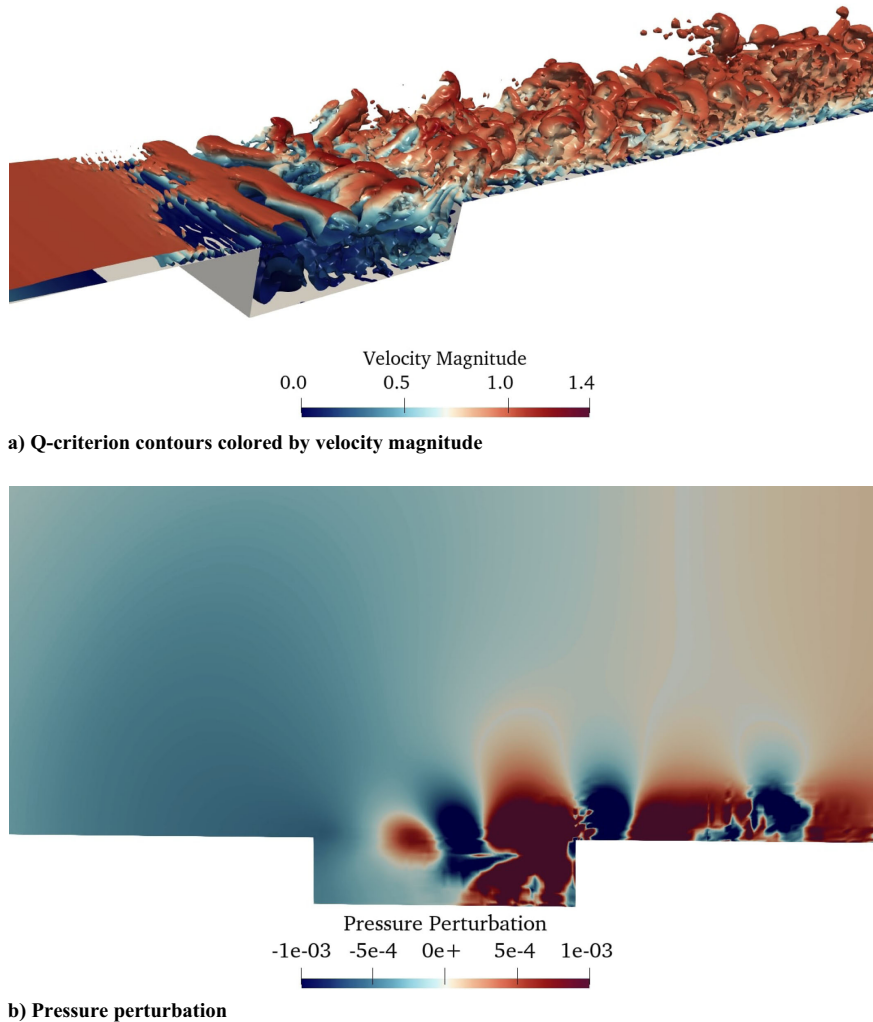


Fig. 8 The Q -criterion contours and pressure perturbation for the baseline design of the open deep cavity.

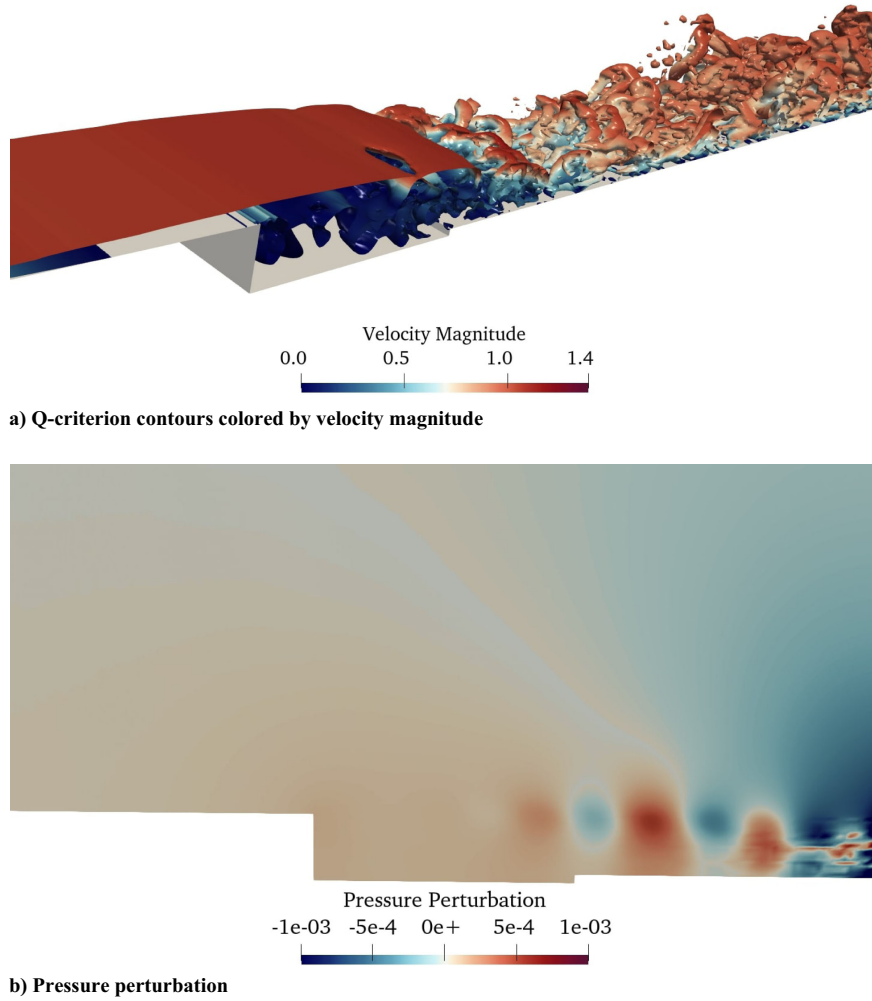


Fig. 9 The Q -criterion contours and pressure perturbation for the optimum design of the open deep cavity.

design parameter space and the convergence of the objective function.

The Q -criterion contours colored by velocity magnitude and the pressure perturbation of both the baseline and optimum designs are shown in Figs. 8 and 9, respectively. Comparing these figures, turbulent structures over the cavity are reduced significantly in the optimum design, and the shear layer expands over the cavity, resulting in much lower noise emission. Furthermore, the power spectral density (PSD) of the OASPL is plotted against the Strouhal number for both the baseline and optimum designs in Fig. 10, which follows the Welch's method of periodograms [47] and involves

dividing the time period into six windows with a 50% overlap. This figure illustrates the OASPL reduction across all frequency ranges.

IV. Tandem Cylinders

The flow around two tandem cylinders consists of multiple flow features, including flow separation, reattachment, recirculation, and quasi-periodic vortex shedding, amongst others. The physics of such flows is highly dependent on the diameter ratio of the cylinders, the spacing between them, and the Reynolds number. The diameter ratio of the cylinders is defined as $r = D_d/D_u$, where D_d and D_u are the downstream and upstream diameter of the cylinders, respectively. The spacing of the cylinders, s , is defined as the distance between the rear of the upstream cylinder and the front of the downstream cylinder. These definitions are depicted in Fig. 11.

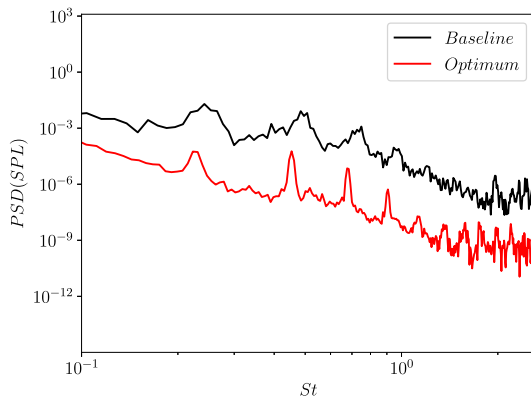


Fig. 10 The sound spectra for the open deep cavity.

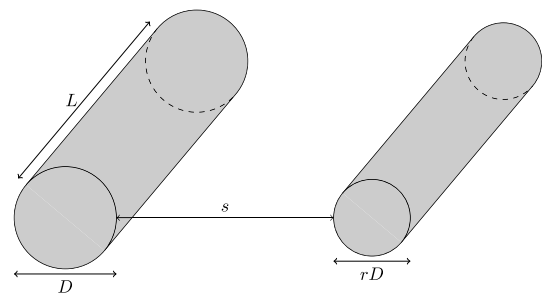


Fig. 11 The geometry of two cylinders in a tandem configuration.

The three-dimensional wake development of a single cylinder was studied by Williamson [48]. Additionally, Papaioannou et al. [49] investigated the three-dimensionality effects of flow over two tandem cylinders, varying Reynolds number, and the spacing distance between the cylinders. They found that as Reynolds number increased, two-dimensional results diverged from three-dimensional ones, especially beyond a critical Reynolds number where wake three-dimensionality initiated. The Reynolds number of our study, based on the upstream cylinder's diameter, is $Re_D = 1000$ because the wake will develop considerable three-dimensionality and this Reynolds number is associated with the early turbulent regime [49].

A. Validation

In this section, the simulation of flow over two tandem cylinders is validated using reference DNS data [49] along with a grid independence study of the time-averaged lift and drag coefficients and OASPL at a near-field observer located $2D$ above the upstream cylinder. Then, the optimization is performed similarly to our previous work [35], where sound at the near-field observer is minimized. The design variables are the ratio of the cylinders' diameters, r , and the distance between the two, s .

1. Computational Details

The cylinders are located at a distance of $s/D = 1$ with a ratio of $r = 1$ and have a spanwise length of $L/D = 10$, following previous studies [49]. The Reynolds number, based on the upstream cylinder's diameter, is $Re_D = 1000$, corresponding to the early turbulent regimes [49], and the Mach number is 0.2. The boundary-layer region extends to $0.5D$ around the cylinders, with the inlet boundary placed $5D$ away from the upstream cylinder and the outlet boundary $55D$ away from the downstream cylinder. The computational domain is extended to $10D$ in the y direction. The stretching ratio for the first $5D$ and $1D$ elements in the x - and y directions, respectively, is 1.05, and that of the remaining elements is 1.075. The smallest element size in the domain is $0.1D$, which is in the boundary-layer region. A total number of 31,780 hexahedral elements are used. The mesh of the tandem cylinders is shown in Fig. 12. Periodic boundary conditions are applied in the spanwise direction, whereas a no-slip boundary condition is imposed on the surface of the cylinders, along with Riemann invariant boundary conditions at the inlet and outlet of the computational domain. The simulation is run for $100t_c$, where $t_c = D/U_\infty$, to allow initial transients to disappear, followed by a subsequent period of $500t_c$ to obtain an average of the statistical quantities.

2. Results and Discussion

The sufficiency of the spanwise length is investigated by computing the correlation coefficient of the velocity fluctuation and the pressure perturbation along the z direction. The correlation plot, demonstrated in Fig. 13, ensures the uncorrelated fluctuations in the z direction at a separation of half of the domain size. Furthermore, the time-averaged drag coefficient and the OASPL at the observer are computed using different averaging window lengths, summarized in Table 2. The time-averaged drag coefficient of the upstream cylinder is computed using $\mathcal{P}2$ and $\mathcal{P}3$ simulations. The $\overline{C}_{D1}$ obtained using the $\mathcal{P}3$ simulation is 0.997, which is in good agreement with the reference value of 0.988 [49]. Table 2 shows that the difference in the statistical time-averaged quantities is

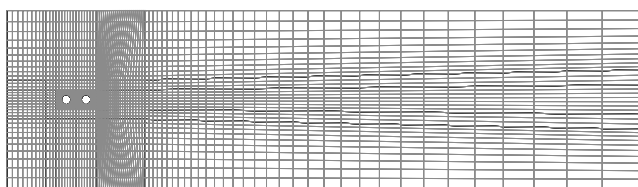


Fig. 12 The mesh of the two cylinders in a tandem configuration.

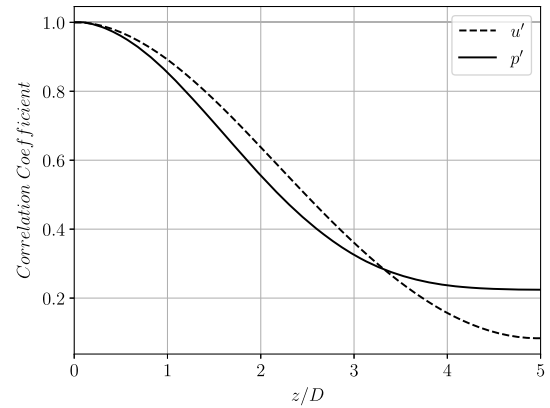


Fig. 13 The correlation coefficient in the spanwise direction for the tandem cylinders.

Table 2 The $\overline{C}_{D1}$ and OASPL at the observer, for the tandem cylinder configuration using different lengths of the averaging window

Averaging window size	$\overline{C}_{D1}$		OASPL, dB	
	$\mathcal{P}2$	$\mathcal{P}3$	$\mathcal{P}2$	$\mathcal{P}3$
$200t_c$	0.962374	0.994465	126.5	125.1
$300t_c$	0.963871	0.994915	126.9	125.2
$400t_c$	0.965569	0.996092	127.3	125.2
$500t_c$	0.966651	0.996752	127.6	125.2
$600t_c$	0.967519	0.997042	127.7	125.3
$700t_c$	0.968142	0.996965	127.8	125.3

negligible beyond $500t_c$. Thus, in this study, the statistical quantities are averaged for $500t_c$.

B. Optimization

The distance between the two cylinders, s , and the ratio between the diameters of the cylinders, r , are the design variables, $\mathcal{X} = [s, r]$. The objective function is $\mathcal{F} = p'_{rms}$ at $2D$ above the upstream cylinder.

1. Results and Discussion

The optimization problem converges in 18 MADS iterations with a total of 48 objective function evaluations. The baseline and optimum designs are shown in Fig. 14. The design space and objective function convergence are shown in Fig. 15, where the optimum design is found as $(s, r) = (2.0291D, 1.7563D)$. The optimization process explores a wide range of design variables, as illustrated in Fig. 15a. Q -criterion contours, colored by velocity magnitude, and acoustic field at the midplane are shown for both the baseline and optimum designs in Figs. 16 and 17, respectively. The optimized design

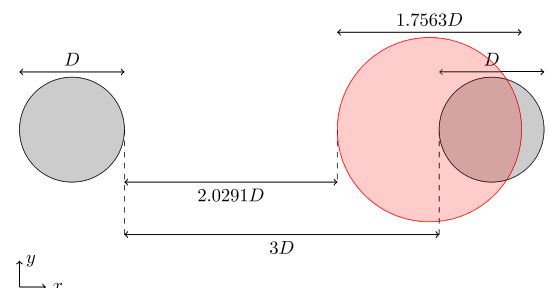


Fig. 14 The baseline, in black, and optimum, in red, designs of the tandem cylinders.

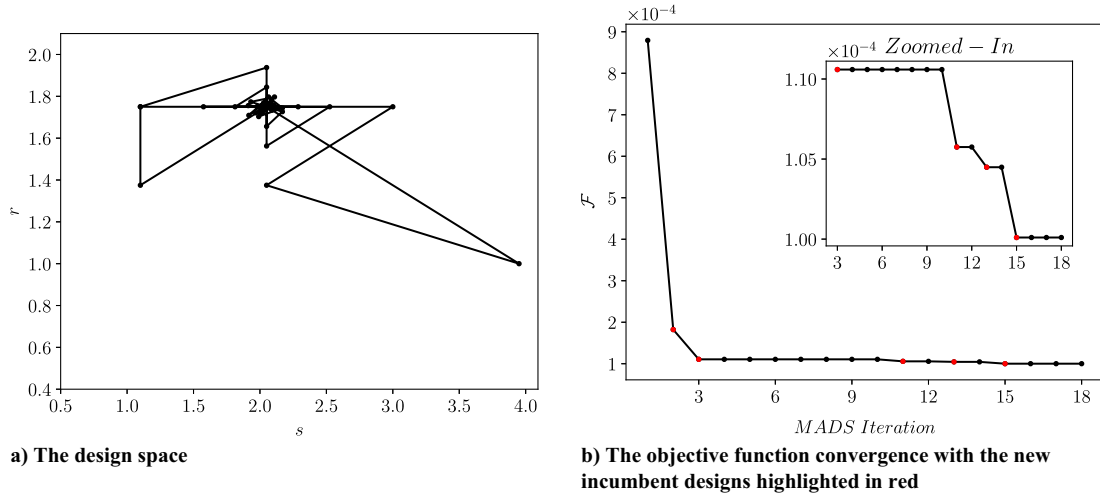


Fig. 15 The design space and objective function convergence for the tandem cylinders.

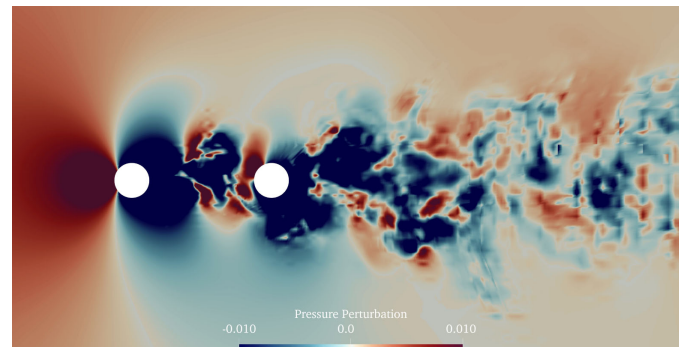
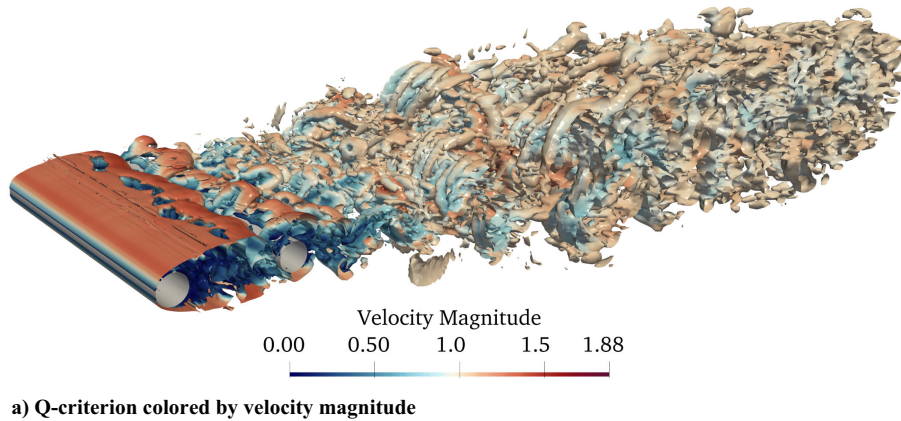


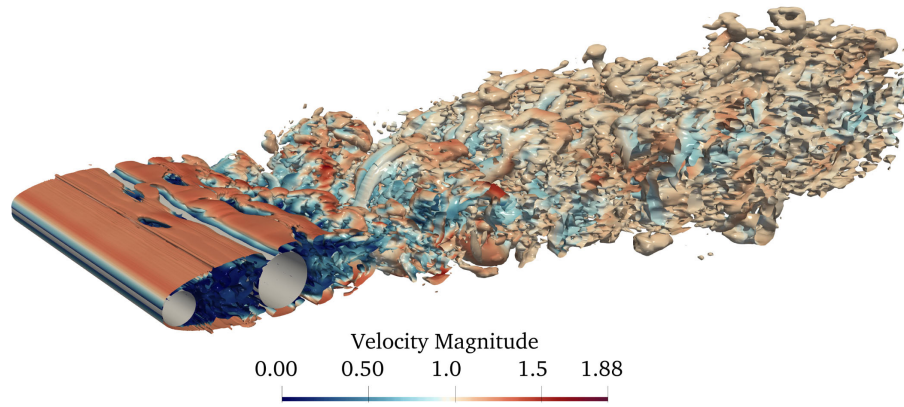
Fig. 16 The baseline tandem cylinder design at $t_c = 600$.

exhibits a smoother flowfield, resulting in reduced noise emissions. The OASPL of the initial design at the observer, 2D above the upstream cylinder, is 125.3 dB, which decreases to 114.1 dB for the optimized configuration. Lastly, Fig. 18 presents the PSD of OASPL vs Strouhal number, computed using Welch's method of periodograms [47] with three windows and a 50% overlap. It is evident that the optimum design displays higher intensity PSD of OASPL over a broad frequency range, while achieving a lower OASPL value, primarily due to a decrease in the largest-magnitude modes. Furthermore, this behavior can be attributed to the baseline design producing high-intensity sound at specific frequencies ($St = 0.63, 0.77$, and 0.90), contributing to its elevated peak

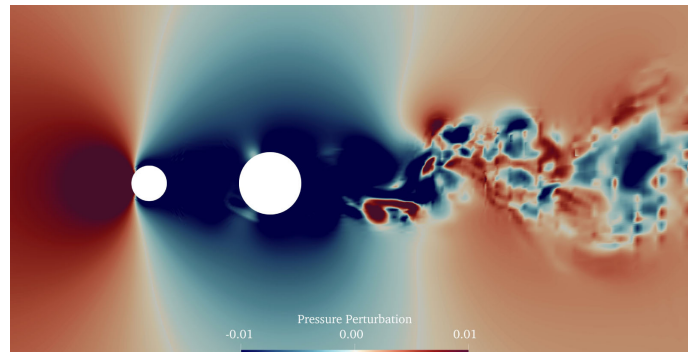
OASPL, whereas the optimum design distributes its energy across a wider frequency spectrum.

V. NACA Four-Digit Airfoil

The flow over NACA four-digit airfoils is investigated in this section. The computational domain, previously used by the authors [35], is extruded in the z direction. The validation of the flow simulation is conducted using an ILES [50] and a grid independence study for a NACA0012 airfoil. Subsequently, four design parameters, akin to those in [35], are selected, and the gradient-free MADS optimization technique is employed. The



a) Q-criterion colored by velocity magnitude



b) Acoustic pressure field at midplane

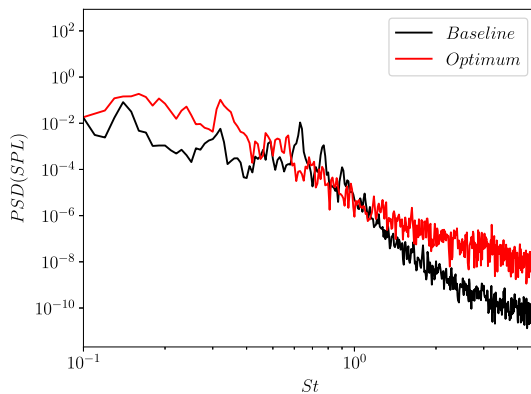
Fig. 17 The optimum tandem cylinder design at $t_c = 600$.

Fig. 18 The sound spectra for the tandem cylinders.

maximal positive basis construction is employed for the optimization algorithm.

A. Validation

Validation for flow over a NACA0012 airfoil at an angle of attack of 6 deg is conducted. The validation process involves comparing the time-averaged lift and drag coefficients obtained from two distinct grid resolutions with those from an ILES [50]. Moreover, the time-averaged pressure coefficient, the skin friction coefficient, and the OASPL at a near-field observer is computed using both grid resolutions and various time-averaging window lengths. This analysis ensures the independence of the results to both grid resolution and time-averaging window lengths. Detailed computational procedures and validation results are presented in the subsequent sections.

1. Computational Details

The computational grid consists of 121,520 hexahedral elements, illustrated in Fig. 19. The domain extends to $20c$ in the x direction,

$10c$ in the y direction, and $0.2c$ in the z direction, with $c = 1$ representing the airfoil chord. Notably, elements in the wake region are inclined at the angle of attack to accurately capture trailing-edge vortices. The flow conditions are characterized by a Reynolds number of 23,000, a freestream Mach number of $M = 0.2$, and Prandtl number of $Pr = 0.71$. The simulation is run for 10 convective times to allow the initial transition to disappear and then is run for another 70 convective times for flow statistics averaging. Additionally, a variable solution polynomial degree is implemented to eliminate acoustic wave reflections from boundaries, as demonstrated in Fig. 20.

2. Results and Discussion

Two distinct grid resolutions are employed with maximum solution polynomial degrees of $\mathcal{P}3$ and $\mathcal{P}4$. The time-averaged lift and drag coefficients are compared to the ILES reference data [50], presented in Table 3. The difference between the time-averaged lift coefficient obtained from the $\mathcal{P}4$ simulation and the reference data is minimal, affirming the adequacy of the $\mathcal{P}4$ simulation's grid resolution. Furthermore, the time-averaged drag coefficient differs by less than 1.3% from the reference data. The OASPL at an observer located two unit chord lengths below the trailing edge is computed for both $\mathcal{P}3$ and $\mathcal{P}4$ simulations. Various averaging window lengths are applied, and the results are summarized in Table 4. It is evident that the $\mathcal{P}$ and $\mathcal{P}3$ simulations differ by only 0.5 dB. The time-averaged pressure coefficient, $\overline{C_p}$, and the skin friction coefficient, C_f , for both resolutions are shown in Figs. 21 and 22, respectively. These plots show that the separation points, identified with each simulation, are very close and differ by less than 2%. Considering the findings presented in Tables 3 and 4, and in Figs. 21 and 22, we opt to conduct $\mathcal{P}4$ simulation for a total duration of 70 convective times for the optimization study.

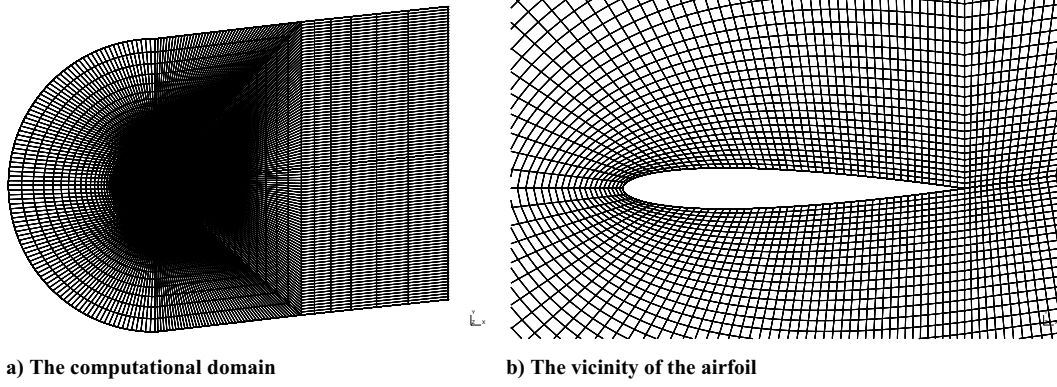


Fig. 19 The computational grid for NACA0012 airfoil at $\alpha = 6$ deg.

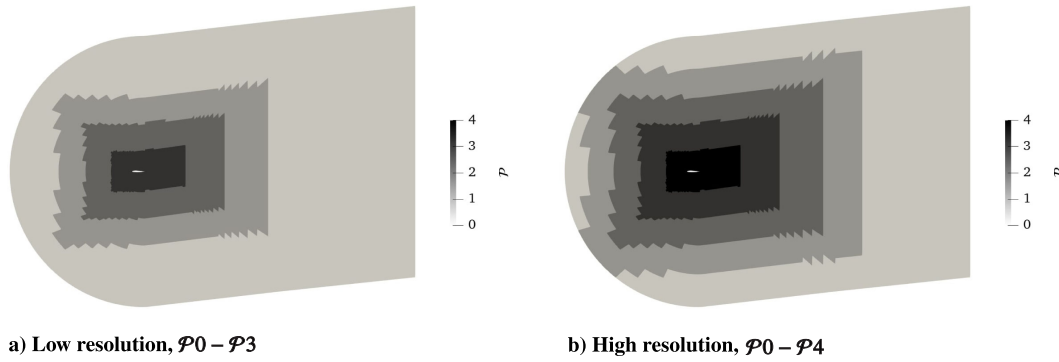


Fig. 20 Different solution polynomial distributions for grid independence study of NACA0012 airfoil at $\alpha = 6$ deg.

Table 3 The time-averaged lift and drag coefficients of NACA0012 airfoil at $\alpha = 6$ deg

	$\mathcal{P}0 - \mathcal{P}3$	$\mathcal{P}0 - \mathcal{P}4$	Reference [50]
$\overline{C_L}$	0.6534	0.6399	0.6402
$\overline{C_D}$	0.0553	0.0548	0.0541

Table 4 The grid independence study of OASPL using different averaging window lengths for NACA0012 airfoil at $\alpha = 6$ deg

Averaging window length	OASPL, dB	
	$\mathcal{P}0 - \mathcal{P}3$	$\mathcal{P}0 - \mathcal{P}4$
$20t_c$	114.9	116.3
$40t_c$	115.7	116.3
$60t_c$	115.7	116.2
$80t_c$	115.7	116.2

B. Optimization

The design parameters are maximum camber $c_{\max}^a$ and its location $x_{c_{\max}^a}$, maximum thickness $t_{\max}^a$, and angle of attack α , that is, $\mathcal{X} = [c_{\max}^a, x_{c_{\max}^a}, t_{\max}^a, \alpha]$. The maximum camber range is set to $c_{\max}^a \in [-10, 10]$ as a percentage of the chord, with the distance from the airfoil leading edge in the range of $x_{c_{\max}^a} \in [4, 9]$ as a 10th of the chord. The maximum thickness of the airfoil is within the range of $t_{\max}^a \in [6, 18]$ as a percentage of the chord. Finally, the angle of attack varies from 0 to 12 deg. The objective function is defined as

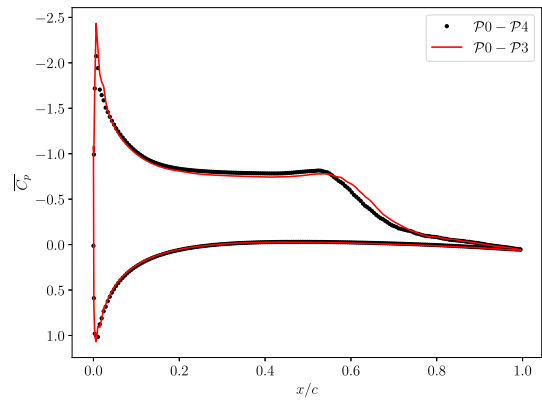


Fig. 21 The time-averaged pressure coefficient for both $\mathcal{P}3$ and $\mathcal{P}4$ simulations.

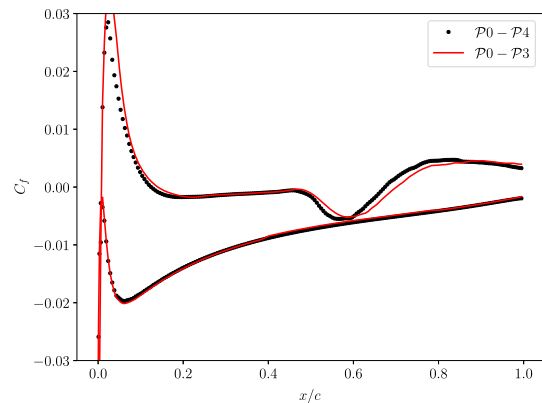


Fig. 22 The skin friction coefficient for both $\mathcal{P}3$ and $\mathcal{P}4$ simulations.

the OASPL at the observer with constraints on both the mean lift and mean drag coefficients. A quadratic penalty term is added to the objective function when the lift coefficient deviates from the baseline design, and an additional quadratic penalty term is added when the mean drag coefficient is above the baseline design. The objective function is defined as

$$\mathcal{F} = \begin{cases} \text{OASPL} + \epsilon_1 (\overline{C_L} - \overline{C_{L,\text{baseline}}})^2 + \epsilon_2 (\overline{C_D} - \overline{C_{D,\text{baseline}}})^2 & \overline{C_D} > \overline{C_{D,\text{baseline}}} \\ \text{OASPL} + \epsilon_1 (\overline{C_L} - \overline{C_{L,\text{baseline}}})^2 & \overline{C_D} \leq \overline{C_{D,\text{baseline}}} \end{cases} \quad (13)$$

where the constants ϵ_1 and ϵ_2 are set to 8000 and 400,000, respectively, to compensate for the order of magnitude difference in OASPL and $\overline{C_L}$ and $\overline{C_D}$. The defined objective function minimizes the OASPL while maintaining the mean lift coefficient, and ensures the optimized airfoil has a similar or lower mean drag coefficient.

1. Results and Discussion

This optimization procedure converges after 22 MADS iterations, consisting of 172 objective function evaluations. The baseline and optimum designs, depicted in Fig. 23, demonstrate that reducing the airfoil thickness correlates with lower noise emission. This aligns with the expectation that moving less air leads to reduced noise levels. However, decreasing the airfoil's thickness raises concerns about structural integrity and increases the risk of flutter. The design space and the convergence of the objective function are shown in Fig. 24. The optimal airfoil design has a maximum camber of $c_{\text{max}}^a = 0.140625\%$ of the chord, at a 6.5 10th of the chord distance from the leading edge, with a thickness of $t_{\text{max}}^a = 8.859375\%$ of the chord, at an angle of attack of $\alpha = 6.28125^\circ$ deg. The OASPL of the optimized airfoil is decreased to 110.6 dB, the mean lift coefficient

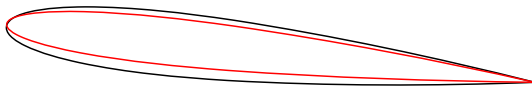
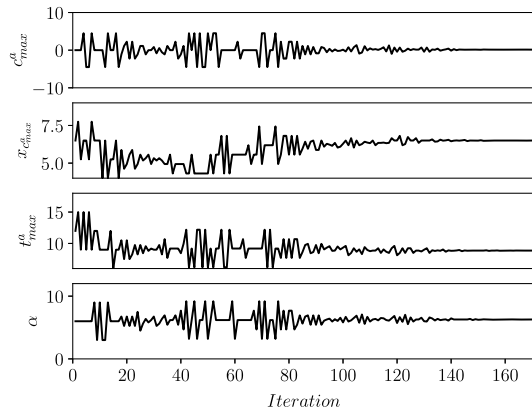


Fig. 23 The baseline, in black, and optimum, in red, designs of the NACA four-digit airfoil.



a) The design space

is $\overline{C_L} = 0.6556$, and finally, the mean drag coefficient is decreased by 7.4% to $\overline{C_D} = 0.0509$.

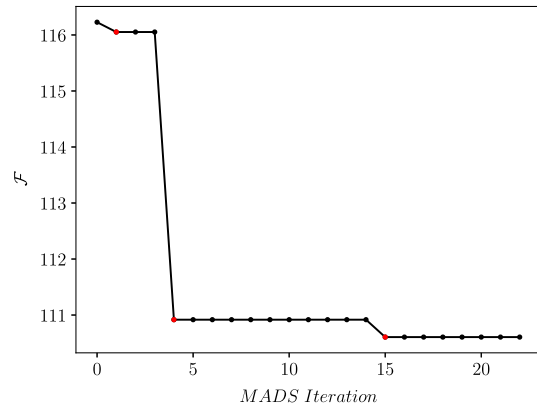
Figures 25 and 26 depict the Q -criterion colored by velocity magnitude and pressure perturbation at midplanes for the baseline and optimum designs. In the optimum design, the separation point shifts toward the leading edge, yielding smaller and less energetic

structures, resulting in reduced noise emission. This leads to a 5.7-dB decrease in OASPL at a near-field observer. Figure 27 presents the PSD of OASPL as a function of the Strouhal number, computed using Welch's method of periodograms [47] with three windows and a 50% overlap. It is evident that the optimum design displays lower-intensity OASPL energy across various frequency ranges.

VI. Conclusions

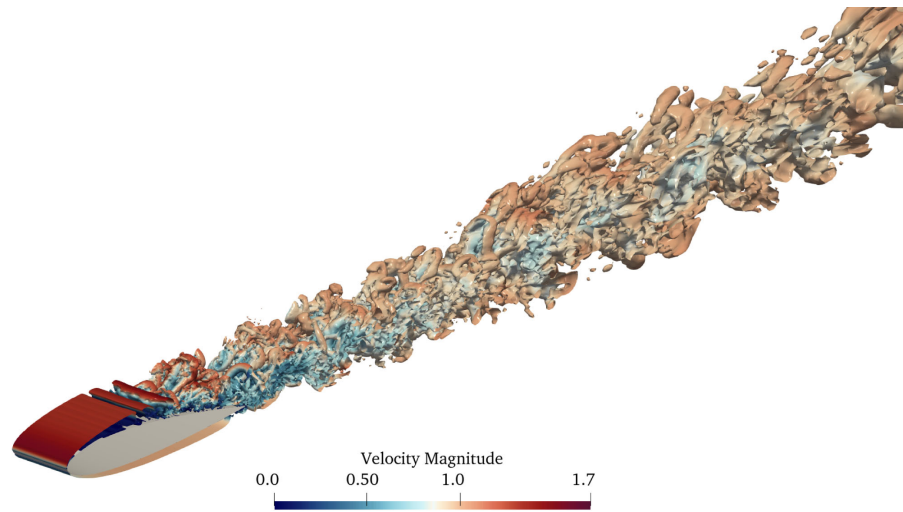
In conclusion, the authors present an aeroacoustic shape optimization framework using the MADS optimization algorithm in conjunction with high-order flux reconstruction spatial discretization and large eddy simulation (LES). This framework effectively reduces overall sound pressure level at a near-field observer. A key contribution of this work is the elimination of runtime dependency on the number of design parameters through parallel implementation. This addresses a key challenge in gradient-free optimization techniques, enhancing the robustness and computational efficiency of our framework. These findings hold significant importance for aeroacoustic shape optimization, with potential applications in the aerospace industry where noise reduction is of paramount importance.

It is important to acknowledge that the current study considers a maximum of four design parameters in the airfoil case. Although the framework demonstrates consistent runtime for each optimization iteration, equivalent to a single computational fluid dynamics simulation with sufficient computational resources, the scalability of this approach to hundreds or thousands of parameters requires further exploration. It should also be noted

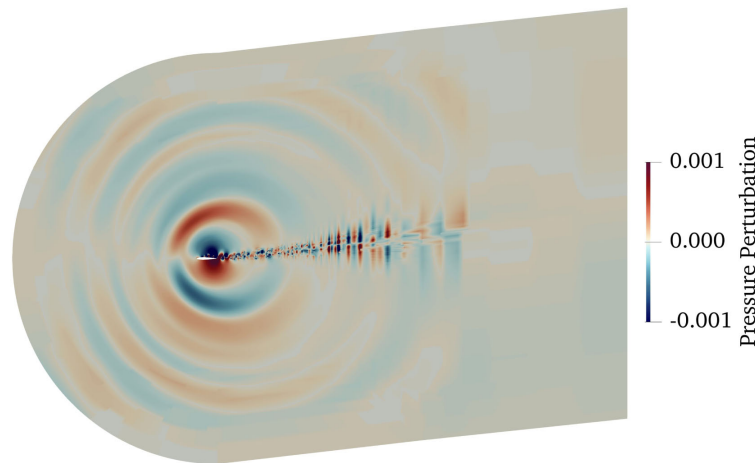


b) The objective function convergence with the new incumbent design highlighted in red

Fig. 24 The design space and objective function convergence of the NACA four-digit airfoil optimization.



a) Q-criterion colored by velocity magnitude



b) Acoustic pressure field at midplane

Fig. 25 The baseline airfoil at $t_c = 70$.

that the computational cost of adjoint-based optimization methods is inherently independent of the number of design parameters, making them an attractive option for high-dimensional problems. However, adjoint methods are known to exhibit instability when applied to LES [51], limiting their applicability in such contexts. Although the proposed framework does not achieve a computational cost that is entirely independent of the number of design parameters, it does facilitate optimization in constant time, assuming adequate parallel resources are available.

The feasibility of the proposed aeroacoustic shape optimization framework can be assessed through testing at higher Reynolds numbers and addressing industry-relevant problems. Additionally, exploring the integration of a far-field acoustic solver into the framework is a promising avenue, potentially broadening its capability to address a more extensive range of aeroacoustic challenges. Furthermore, future work can focus on integrated sound pressure levels across observer surfaces rather than discrete points, aligning the framework more closely with practical applications. This research suggests the potential for more efficient aeroacoustic shape optimization methods, with notable implications for quieter and more efficient aerodynamic designs. Moreover, incorporating design limits for optimization constraints, such as structural failure and flutter phenomenon in airfoil optimization, presents a crucial consideration for enhancing the

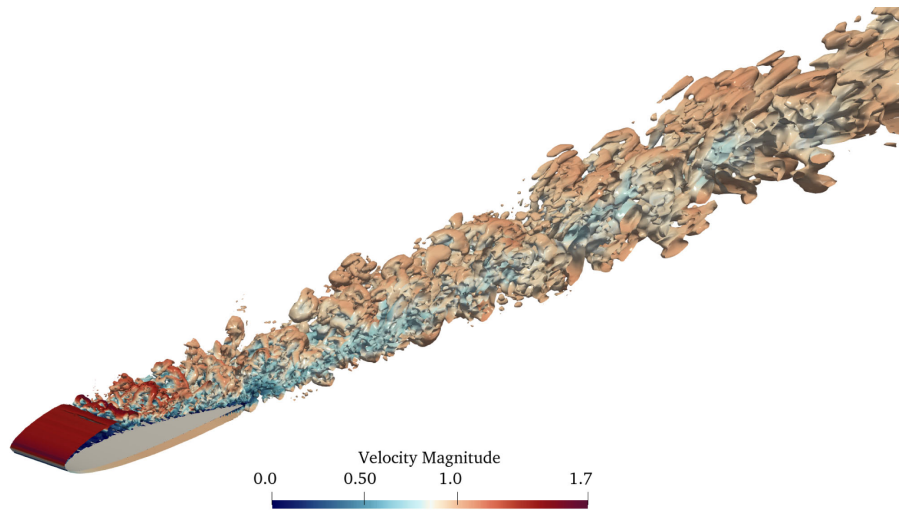
robustness and applicability of the framework in real-world engineering scenarios.

Data Availability Statement

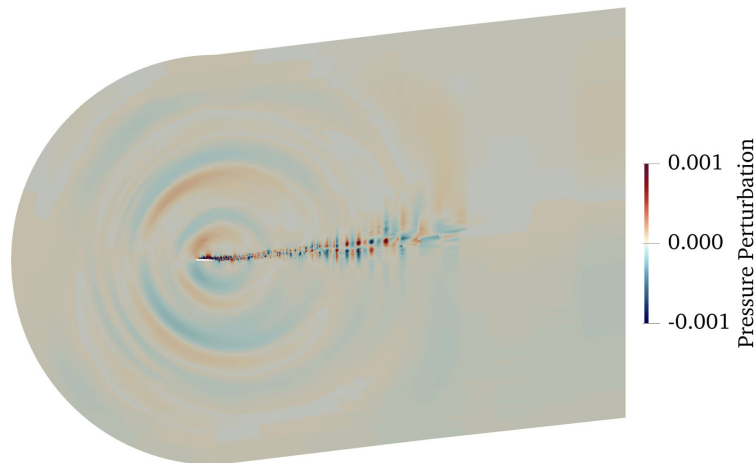
Data relating to the results in this paper can be downloaded from the publication's website under a CC-BY-NC-ND 4.0 license.

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a) Q-criterion colored by velocity magnitude



b) Acoustic pressure field at midplane

Fig. 26 The optimum airfoil at $t_c = 70$.

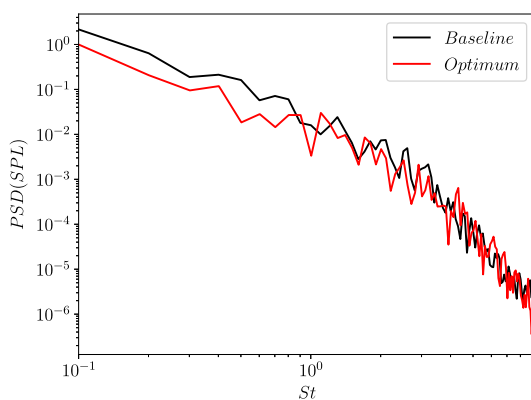


Fig. 27 The sound spectra for the NACA four-digit airfoils.

have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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